MEM6810 Engineering Systems Modeling and Simulation

Sino-US Global Logistics Institute
Shanghai Jiao Tong University

Spring 2024 (full-time)

Assignment 2

Due Date: April 23 (in class)

Instruction

(a) You can answer in English or Chinese or both.
(b) Show **enough** intermediate steps.
(c) Write your answers **independently**.
(d) If you copy the solutions from somewhere, you must **indicate the source**.

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Question 1 (10 points)

There are three definitions for Poisson process (See Lec 3 pages 15-16/64). Prove that, if a stochastic process satisfies Definition 3, then is must satisfy Definition 1.

Question 2 (15 points)

For a Poisson process $\{N(t),\ t \geq 0\}$ with rate λ, given that $N(t) = n$, the n arrival times $S_1, \ldots, S_n$ have the same distribution as the order statistics corresponding to n independent RVs uniformly distributed on the interval $(0, t)$. (See Lec 3 page 20/64.) Prove it rigorously.

Hint: If RVs Y_i, $i = 1, \ldots, n$, are uniformly distributed over $(0, t)$, then the joint pdf of the order statistics $Y_{(1)}, Y_{(2)}, \ldots, Y_{(n)}$ is

$$f(y_1, y_2, \ldots, y_n) = \frac{n!}{t^n}, \quad 0 < y_1 < y_2 < \cdots < y_n < t.$$

Question 3 ($10 + 15 = 25$ points)

For an $M/M/\infty$ queue, derive its limiting (steady-state) distribution (see Theorem 6 on Lec 3).

(1) Use the result of $M/M/s$ queue and let $s \to \infty$.
(2) Use the state space diagram.

Question 4 (20 points)

For an $M/M/s/K$ queue, derive its limiting (steady-state) distribution (see Theorem 8 on Lec 3) using the state space diagram.

Question 5 $(4+4+2=10 \text{ points})$

Consider an $M/M/1/5$ queue with arrival rate $\lambda = 10/\text{hour}$ and service rate $\mu = 15/\text{hour}$.

(1) What is the probability that an arrival customer finds the station is full?

(2) What is the expected amount of time a customer who enters the station will spend in it?

(3) What is the expected amount of time an arrival customer will spend in the station? (Note: If a customer doesn't enter the station, the amount of time he spends in the station is 0.)

Question 6 (10 points)

Consider a Jackson queueing network with external arrival rate $\boldsymbol{\lambda} = [\lambda_1, \lambda_2, \lambda_3]^\intercal = [6, 2, 4]^\intercal$, service rate $\boldsymbol{\mu} = [\mu_1, \mu_2, \mu_3]^\intercal = [6, 4, 12]^\intercal$, server number $\boldsymbol{s} = [s_1, s_2, s_3]^\intercal = [2, 3, 1]^\intercal$, and routing matrix

$$\boldsymbol{P} = \begin{bmatrix} 0 & 0.6 & 0.2 \\ 0 & 0 & 0.4 \\ 0 & 0.5 & 0.1 \end{bmatrix}.$$

Calculate the expected number of customers in the entire network in steady state.

Question 7 $(2+8=10 \text{ points})$

If a machine produces some products one by one. The time length of producing one piece of product follows $\text{Exp}(a)$. The finished products are stored in a warehouse, whose capacity limit is b pieces of products. When the warehouse is full, the machine will pause; once there is available space in the warehouse, the machine will continue the previous job immediately. Customers arrive to the warehouse following a Poisson process with rate c. Each customer will take away one piece of product (ignoring the time length of picking the product). If an arriving customer finds the warehouse is empty, he/she leaves immediately.

(1) If we are interested in the number of products in the warehouse, can this problem be represented by one of the queueing models introduced in Lec 3? (Only need to answer Yes or No.)

(2) If Yes, write down the queueing model and the corresponding parameters, and explain the reason; If No, add or modify some assumptions of this problem so that it can be represented by one of the queueing models introduced in Lec 3, and write down the queueing model and the corresponding parameters.